January 6th, 2013 Name (Please Print) ____________________

Differential Equations - Back Paper Exam - Semester II 13/14

Your Signature ____________________

1. Solve the following Bernoulli equation given by

$$\frac{dx}{dt}(t) - \frac{1}{3}x(t) = tx(t)^4,\ t > 0 \text{ and } x(0) = -2.$$

2. Consider the second order linear differential equation:

$$3\frac{d^2y}{dt^2} - 15\frac{dy}{dt} + 18y = 0$$

   (a) Find the general solution.

   (b) Find the particular solution satisfying $y(0) = 0, y'(0) = \alpha$.

   (c) When is $\lim_{t\to\infty} y(t) = 0$ ?

3. Solve the following short answer questions:

   (a) Let $f : [0, \infty) \times \mathbb{R} \to \mathbb{R}$ be a continuous function which is Lipschitz continuous function in $x$ uniformly in $t$. Prove or Disprove: The initial value problem

$$\frac{dy}{dt} = f(t, y) \text{ for } t > 0, y(0) = 0.$$

   has a unique solution $y : [0, \infty) \to \mathbb{R}$.

   (b) Consider the second order linear differential equation:

$$\frac{d^2y}{dt^2} + p(t)\frac{dy}{dt} + q(t)y = 0,$$

   Suppose $p(t) = \frac{1}{t}, q(t) = \frac{1}{t^2}$ having one regular singularity at 0. Write out the indical equation and possible Frobenius series solutions.

4. Solve the following boundary value problem:

$$\begin{aligned} u_{tt} &= u_{xx} \text{ if } 0 < x, 0 \leq t \\ u_x(0,t) &= 0 \text{ if } t \geq 0 \\ u(x,0) &= x^2e^{-x} \text{ if } 0 < x \end{aligned}$$

5. Solve the Dirichlet Problem given by

$$\begin{aligned} \Delta u &= 0 \text{ if } 0 \leq x \leq 4, 0 \leq y \leq 3 \\ u(0,y) = u(4,y) &= 0 \text{ if } 0 \leq y \leq 3 \\ u(x,0) &= 0 \text{ if } 0 \leq x \leq 4 \\ u(x,3) &= 100 \text{ if } 0 \leq x \leq 4 \end{aligned}$$

(you may assume that the problem has a unique solution).